Benford`s Law

This “Law” is focused on the “relative rate” of figures at the beginning of numbers

Benford`s Law

Naturally there are more small things than big things !!

In the whole world there exist
more sand as pebbles
more pebbles as stones
more stones as rocks
more rocks as mountains
more mountains as mountain-ranges

Benford`s Law

Frank Benford

Physiker, USA

Benford`s Hypothese:
1920

There exist more numbers with a small figure than with a high figure (at the beginning)

Benford`s Law

Benford did more than 20.000 single checks to proof his thesis!:

- Results of the American Baseballleagues
- air pressure figures
- weight of atoms
- Invoices of energy on Solomon Islands
- articles of Reader`s Digest
- numbers of inhabitants of countries, cities …
- bookkeeping of enterprises

....

Result:

The „1“ was always the leading figure
the „9“ always the last one

Benford`s Law

The spread-rates for the “first figure” were

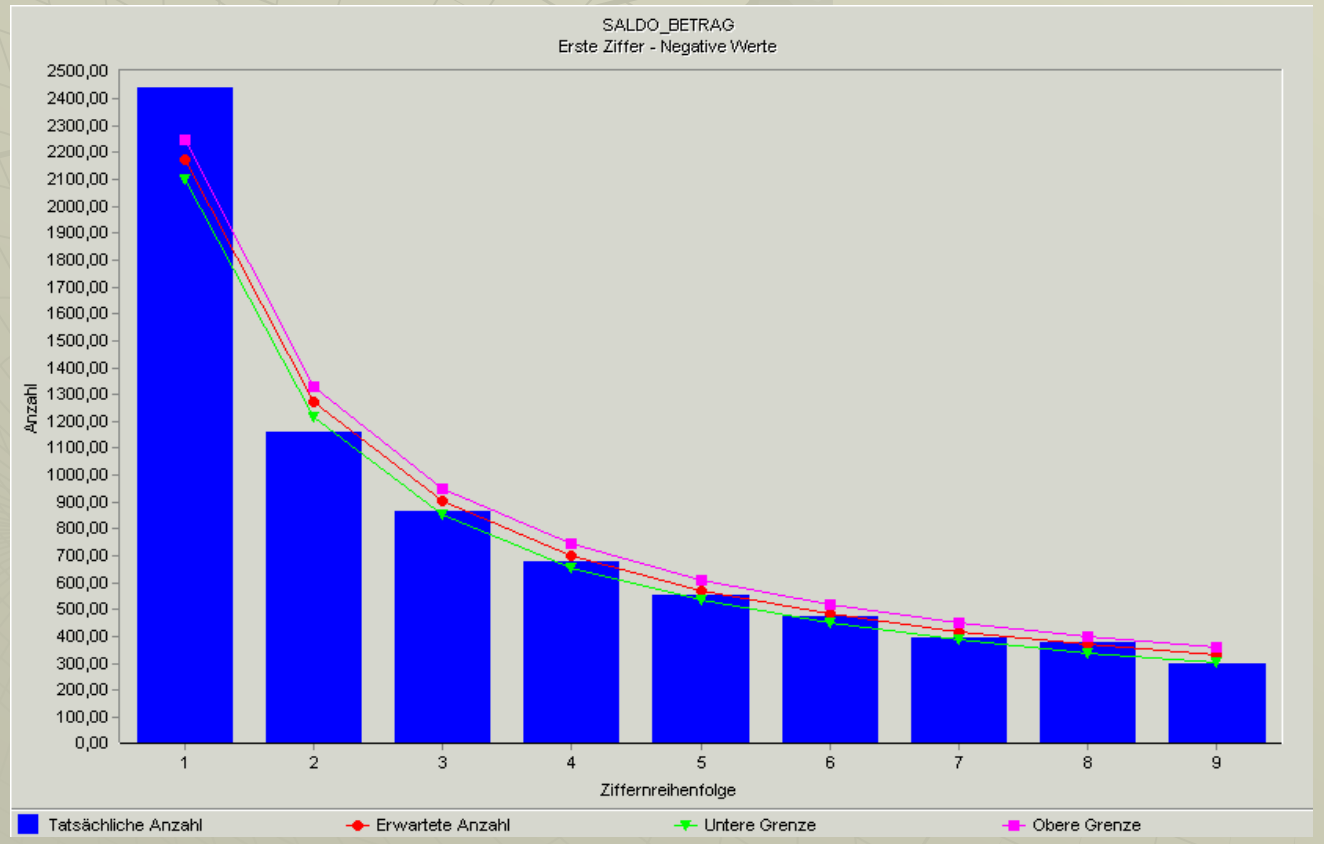
Figure	Rate
1	30,10 %
2	17,60 %
3	12,49 %
4	9,69 %
5	7,91 %
6	6,69 %
7	5,79 %
8	5,11 %
9	4,57 %

Benford`s Law:

every 3. number starts with a „1“

Benford`s Law

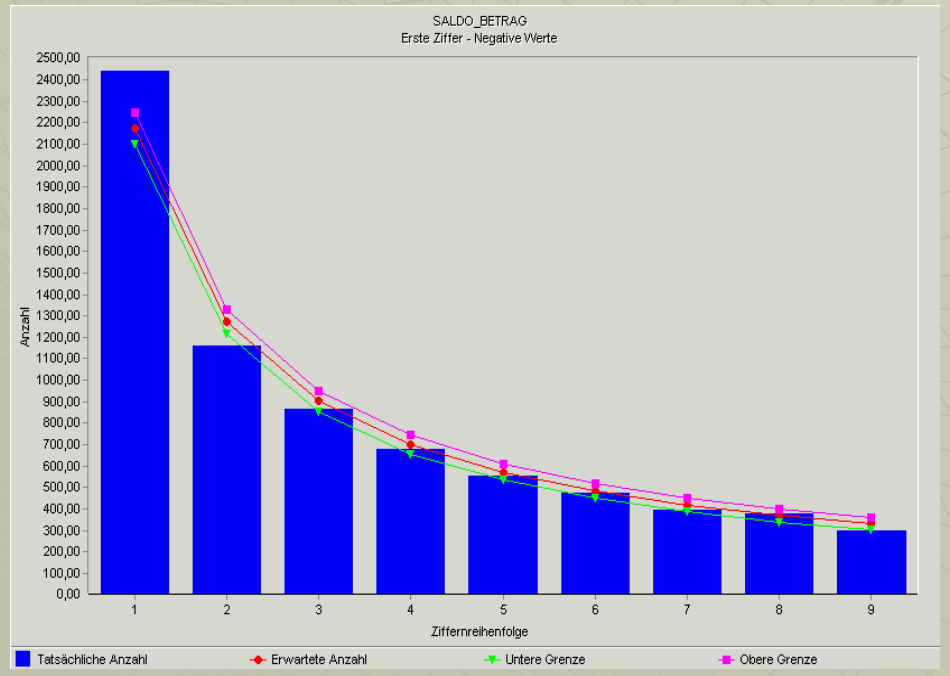
Grafik: 1. Rate-Test



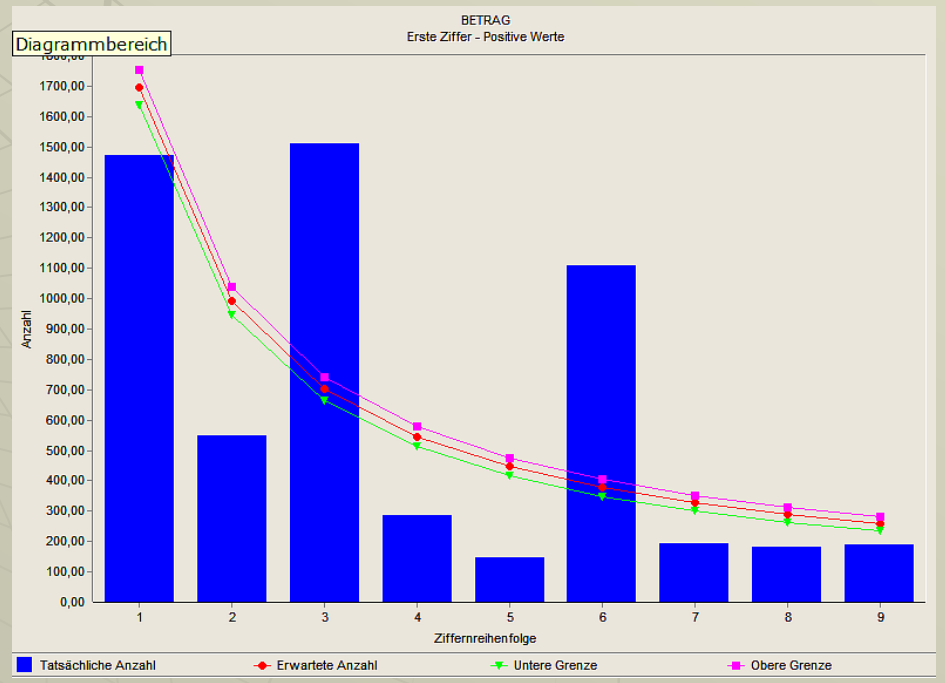
Benford`s Law

Grafik: 1. Rate-Test

normal



noticeable



Is Benford`s law always right ?

law of maths
or
freak of nature

Example Cars

Benford`s Law

Benford`s Law is always right for naturally formed figures / databases

Figures, following the nature-laws as:

Numbers of inhabitants
Length of all rivers of the world
Size / amount of water in all seas/lakes/oceans
Stock-courses
Booking numbers
Turnover
Trading prices

Relevanz of Units

Benford`s Law is universal

Change of Units doesn’t influence the rate!

Km – Miles– Morgen – Zoll – Inch – Acr
EUR - BAT- $ - Yen

Benford`s Law

The Benford check is an own automatic tool within the analyse-programs IDEA/ACL (and others)

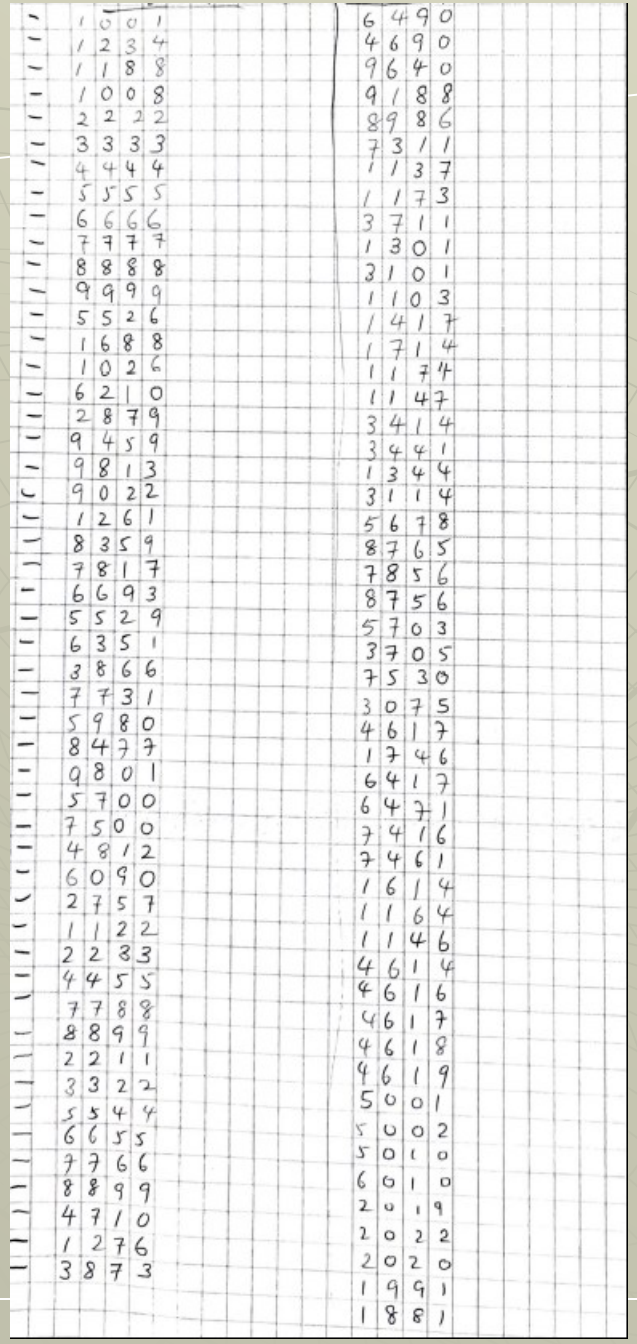
After import of databases you can run the check automatic by allocating the source – “a thing of seconds”

BUT BENFORD-CHECK IS NOT THE PROOF OF THE FRAUD IT’S THE ENTRY FOR FURTHER CHECKS

Benford`s Law

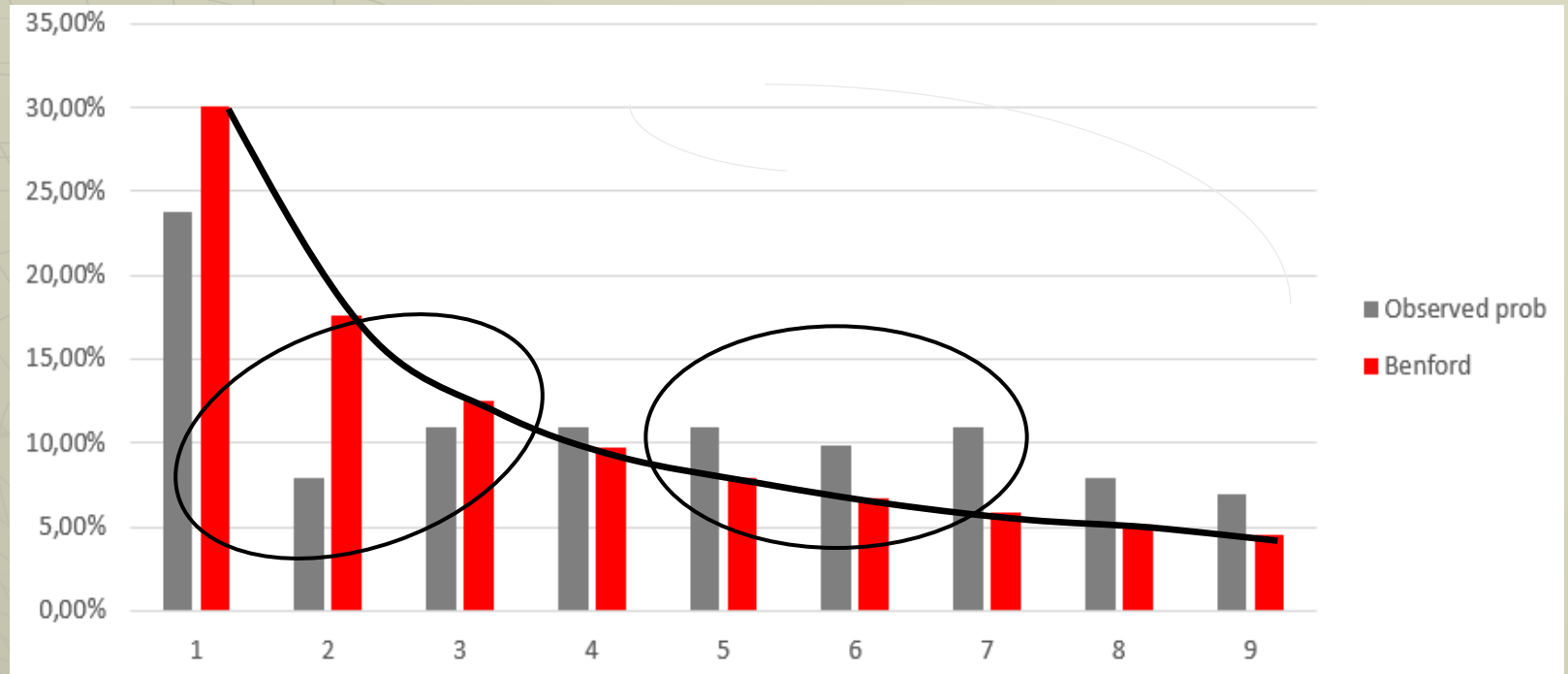
Examples

Wai San Loh



1	1001	51	6490	101	1881
2	1234	52	4690		
3	1188	53	9640		
4	1008	54	9188		
5	2222	55	8986		
6	3333	56	7311		
7	4444	57	1137		
8	5555	58	1173		
9	6666	59	3711		
10	7777	60	1301		
11	8888	61	3101		
12	9999	62	1103		
13	5526	63	1417		
14	1688	64	1714		
15	1026	65	1174		
16	6210	66	1147		
17	2879	67	3414		
18	9459	68	3441		
19	9813	69	1344		
20	9022	70	3114		
21	1261	71	5678		
22	8359	72	8765		
23	7817	73	7856		
24	6693	74	8756		
25	5529	75	5703		
26	6351	76	3705		
27	3866	77	7530		
28	7731	78	3075		
29	5980	79	4617		
30	8477	80	1746		
31	9801	81	6417		
32	5700	82	6471		
33	7500	83	7416		
34	4812	84	7461		
35	6090	85	1614		
36	2757	86	1164		
37	1122	87	1146		
38	2233	88	4614		
39	4455	89	4616		
40	7788	90	4617		
41	8899	91	4618		
42	2211	92	4619		
43	3322	93	5001		
44	5544	94	5002		
45	6655	95	5010		
46	7766	96	6010		
47	8899	97	2019		
48	4710	98	2022		
49	1276	99	2020		
50	3873	100	1991		

Benford`s Law



Results

Testing goodness of fit to Benford's law

Null hypothesis: data is described by Benford's law

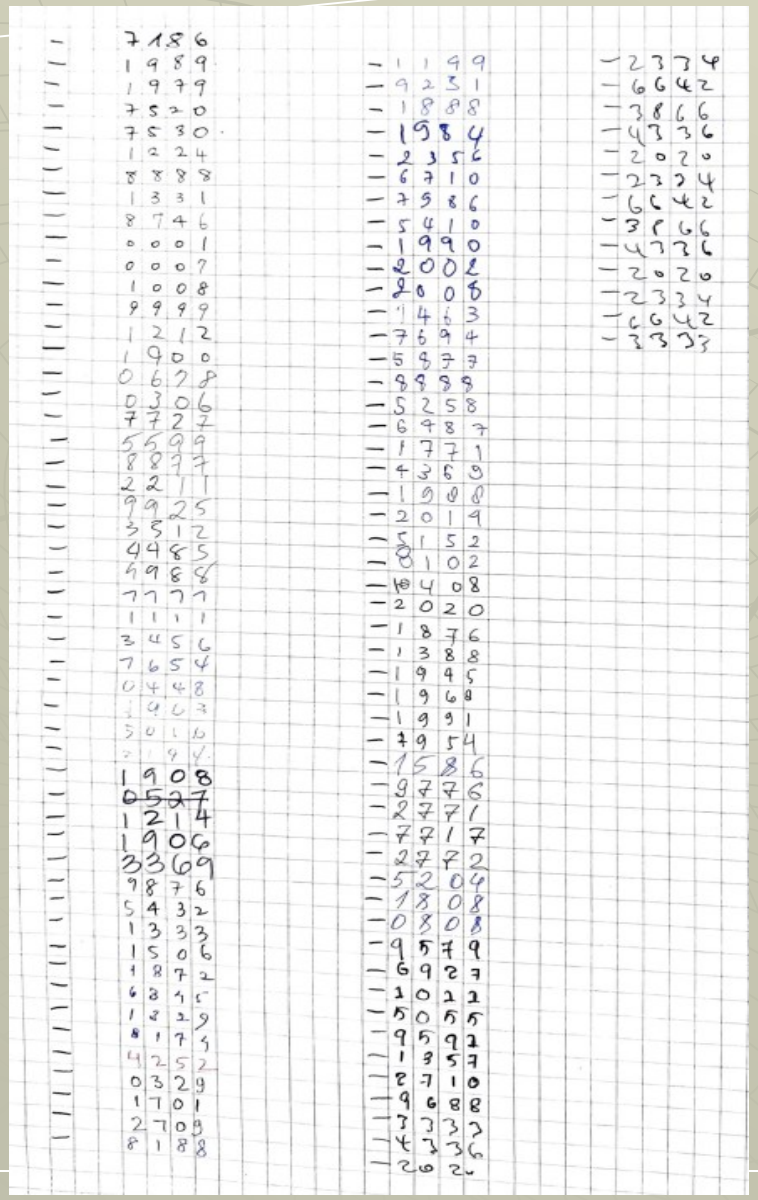
I am more than 95% sure there is something wrong

Benford`s Law

Number	Count	Observed prob	Benford	Chi square values
1	24	23,76%	30,10%	1,3488868
2	8	7,92%	17,61%	5,3837113
3	11	10,89%	12,49%	0,2076704
4	11	10,89%	9,69%	0,1500993
5	11	10,89%	7,92%	1,1274012
6	10	9,90%	6,69%	1,5509684
7	11	10,89%	5,80%	4,5155688
8	8	7,92%	5,12%	1,5541295
9	7	6,93%	4,58%	1,2241097
Number of Data Points	101			
ChiSquare Value	17,063			
Critical Thresholds for ChiSquare Tests (8 degrees of freedom)				
95 percent	15,5073			
99 percent	20,0902			

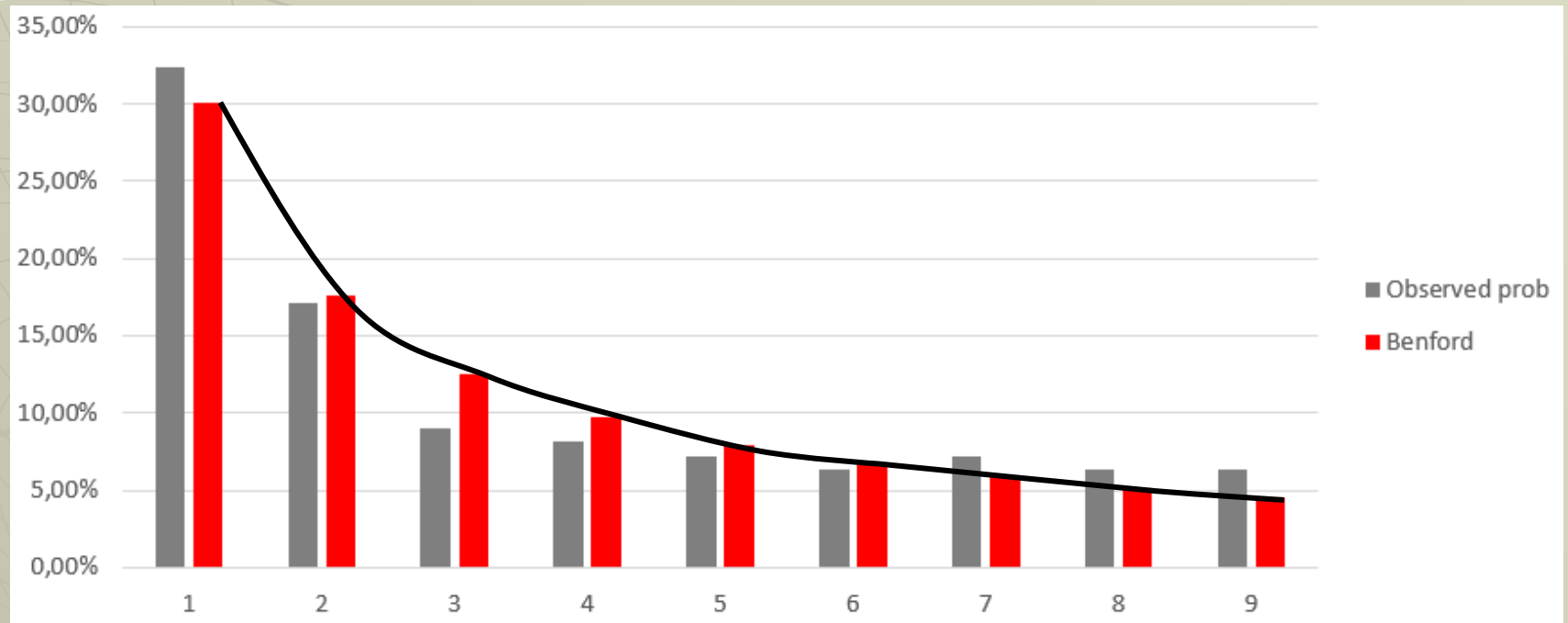
Group

Benford`s Law



1	7186	50	8188	99	4336
2	1989	51	1199	100	2020
3	1979	52	9231	101	2334
4	7520	53	1888	102	6642
5	7530	54	1984	103	3866
6	1224	55	2356	104	4336
7	8888	56	6710	105	2020
8	1331	57	7986	106	2334
9	8746	58	5410	107	6642
10	1001	59	1990	108	3866
11	2007	60	2002	109	4336
12	1008	61	2008	110	2020
13	9999	62	1463	111	2334
14	1212	63	7694	112	6642
15	1900	64	5877	113	3333
16	678	65	8888		
17	306	66	5258		
18	7727	67	6787		
19	5599	68	1771		
20	8877	69	4369		
21	2211	70	1900		
22	9925	71	2019		
23	3312	72	5152		
24	4485	73	8102		
25	4988	74	1408		
26	1111	75	2020		
27	1111	76	1876		
28	3456	77	1388		
29	1654	78	1945		
30	448	79	1968		
31	1963	80	1991		
32	5016	81	7954		
33	2194	82	1586		
34	1908	83	9776		
35	1214	84	2771		
36	1906	85	7717		
37	3369	86	2772		
38	9876	87	5204		
39	5432	88	1808		
40	1333	89	808		
41	1503	90	9579		
42	1872	91	6927		
43	6345	92	1022		
44	1329	93	5055		
45	8179	94	9597		
46	4252	95	1357		
47	329	96	2710		
48	1701	97	9866		
49	2709	98	3333		

Benford`s Law



Results

Testing goodness of fit to Benford's law

Null hypothesis: data is described by Benford's law

Cannot reject Null Hypo	I am not 95% sure there is something wrong
Cannot reject Null Hypo	I am not 99% sure there is something wrong

Benford`s Law

Number	Count	Observed prob	Benfor	Chi square values
1	36	32,43%	30,10%	0,2000846
2	19	17,12%	17,61%	0,0152592
3	10	9,01%	12,49%	1,0789410
4	9	8,11%	9,69%	0,2869839
5	8	7,21%	7,92%	0,0708498
6	7	6,31%	6,69%	0,0250087
7	8	7,21%	5,80%	0,3794620
8	7	6,31%	5,12%	0,3078356
9	7	6,31%	4,58%	0,7264952
Number of Data Points	**111**			
ChiSquare Value	**3,091**			
Critical Thresholds for ChiSquare Tests (8 degrees of freedom)				
95 percent	15,5073			
99 percent	20,0902			

Vertikal (Wert) Achse